

MODAL STRAIN ENERGIES IN COSMIC NASTRAN

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SUMMARY

A computer program was developed to take a NASTRAN output file from a normal modes analysis and calculate the modal strain energies of selected elements. The FORTRAN program can determine the modal strain energies for CROD, CBAR, CELAS, CTRMEM, CQDMEM2, and CSHEAR elements. Modal strain energies are useful in estimating damping in structures.

INTRODUCTION

This work was initiated to predict damping in a large passively damped truss structure. The twelve meter truss structure is currently undergoing modal testing in preparation for controls experiments. An estimate of the total damping in the structure is needed for the controls experiments.

The starting point for the computer program was the ANALYZE program (ref. 1). First, the information needed for the strain energies was extracted from a NASTRAN output file (ref. 2). Element information is extracted from the echo of the bulk data and eigenvector information is extracted from the eigenvector tables for each mode. The element stiffness matrices are formed and then multiplied by the appropriate element eigenvector and its transpose and divided by two. The result is the element strain energies for a given mode. With this information, the modal strain energy method can be used to predict the damping in a viscoelastically damped structure. COSMIC NASTRAN can output element strain energies but only for static analyses. To predict the structural damping, the element modal strain energies for a normal modes analysis have to be found.

SYMBOLS

$w_x = x$ displacements in the plane of the plate in the local coordinate system

$w_y = y$ displacements in the plane of the plate in the local coordinate system

$a_1, b_1, c_1, a_2, b_2, c_2 =$ six undetermined coefficients

$x_1, y_1, \dots, x_3, y_3 =$ coordinates of the 3 nodes of the triangle in the local coordinate system

$\eta =$ shape matrix

$\sigma =$ stress vector

$\epsilon =$ strain vector

G = shear modulus

E = modulus of elasticity

k = element stiffness matrix

ϕ^r = element eigenvector for the r th mode

ELEMENT FORMULATION

As mentioned previously, the formulation of the elements comes from the ANALYZE program. The element stiffness matrices are exactly the same as the COSMIC NASTRAN formulation for the CELAS, CBAR, and CROD elements. The formulation for the CTRMEM and CQDMEM2 elements is slightly different than COSMIC NASTRAN. The CSHEAR formulation is very different from the COSMIC NASTRAN formulation. The basis for the derivation of the shear panel is empirical but accurately constructed finite element models produce satisfactory results. The modal strain energy program will produce good results if the shear panel planform is as close to rectangular as possible. The less skewing of the element, the better the results will be.

The triangular membrane element used in this program is a constant strain plate element. The quadrilateral membrane and shear elements are constructed of four (non-overlapping) of the constant strain triangular membrane elements mentioned above. The elements are assumed to be flat plates which means the warping in the elements is ignored. The elements have a fictitious interior node which is later removed by static condensation. Only shear energy is considered in the stiffness of the shear element where the quadrilateral membrane element considers all the energy in the element.

Since the triangular membrane element is the basis for all the other plate elements in this program, the derivation will be given along with how these triangle elements are used to formulate the quadrilateral and shear elements. The linear displacement field in the triangular element can be represented by

$$w_x = a_1 x + b_1 y + c_1 \quad (1)$$

$$w_y = a_2 x + b_2 y + c_2 \quad (2)$$

or in matrix form

$$\underline{w} = \begin{pmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \end{pmatrix} \quad (3)$$

The six unknown coefficients can be uniquely determined by the six boundary conditions

at the nodes.

$$\begin{pmatrix} v_1 \\ v_3 \\ v_5 \\ v_2 \\ v_4 \\ v_6 \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & 1 & | & 0 & 0 & 0 \\ x_2 & y_2 & 1 & | & 0 & 0 & 0 \\ x_3 & y_3 & 1 & | & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & | & x_1 & y_1 & 1 \\ 0 & 0 & 0 & | & x_2 & y_2 & 1 \\ 0 & 0 & 0 & | & x_3 & y_3 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ b_1 \\ c_1 \\ a_2 \\ b_2 \\ c_2 \end{pmatrix} \quad (4)$$

The inversion of the partitioned diagonal matrix involves simply the inversion of the component matrix. The shape matrix $\underline{\eta}$ is given by

$$\underline{\eta} = \underline{x} \underline{Z}^{-1} \quad (5)$$

where the matrix \underline{x} is given by

$$\underline{x} = \begin{pmatrix} x & y & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x & y & 1 \end{pmatrix} \quad (6)$$

and the \underline{Z} matrix is given by

$$\underline{Z} = \begin{pmatrix} \underline{X} & 0 \\ 0 & \underline{X} \end{pmatrix} \quad (7)$$

The coordinate matrix \underline{X} is given by

$$\underline{X} = \begin{pmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{pmatrix} \quad (8)$$

From linear strain-displacement relations, the strains can be written as

$$\epsilon_x = \frac{\partial w_x}{\partial x} = a_1 \quad (9)$$

$$\epsilon_y = \frac{\partial w_y}{\partial y} = b_2 \quad (10)$$

$$\epsilon_{xy} = \frac{\partial w_x}{\partial y} + \frac{\partial w_y}{\partial x} = b_1 + a_2 \quad (11)$$

From the principle of virtual work, the elements of the member stiffness matrix can be written as

$$k_{ij} = \int_V \sigma^{(i)t} \epsilon^{(j)} dV = \int_V \epsilon^{(i)t} E \epsilon^{(j)} dV \quad (12)$$

where $\sigma^{(i)}$ and $\epsilon^{(j)}$ are the stress and strain matrices corresponding to the unit displacement modes explained in equation 8. Since the linear displacement relation implies constant strain, the integral in equation 12 can be replaced by the volume of the element:

$$k_{ij} = \frac{1}{2} |\underline{X}| \epsilon^{(i)t} E \epsilon^{(j)} \quad (13)$$

where $|X|$ is the determinant of the nodal coordinate matrix which represents twice the area of the element and t is the thickness of the element. Finally, the stiffness matrix of the triangular membrane element is given by

$$k = \frac{1}{2} |X| t \begin{pmatrix} \xi^{(1)t} & E_{\xi}^{(1)} & \xi^{(1)t} & E_{\xi}^{(2)} & \dots & \xi^{(1)t} & E_{\xi}^{(6)} \\ \xi^{(2)t} & E_{\xi}^{(1)} & \xi^{(2)t} & E_{\xi}^{(2)} & \dots & \xi^{(2)t} & E_{\xi}^{(6)} \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ \xi^{(6)t} & E_{\xi}^{(1)} & \xi^{(6)t} & E_{\xi}^{(2)} & \dots & \xi^{(6)t} & E_{\xi}^{(6)} \end{pmatrix} \quad (14)$$

Equation 14 gives the formulation for the stiffness matrix of the triangular membrane elements. What follows, is how four of these triangular elements are used to construct quadrilateral membrane and shear elements.

The stiffness matrix of the quadrilateral membrane element is determined by breaking it into four component triangles. The fictitious node in the quadrilateral is located by averaging the coordinates of the four nodes as follows

$$x_5 = \frac{x_1 + x_2 + x_3 + x_4}{4} \quad (15)$$

$$y_5 = \frac{y_1 + y_2 + y_3 + y_4}{4} \quad (16)$$

The stiffness of the four triangles can then be computed by equation 14. Addition of the four stiffness matrices gives a 10×10 stiffness matrix with two degrees of freedom included for the fifth node. The force displacement relations of the five node quadrilateral are written as

$$R_Q = k_Q r_Q \quad (17)$$

where the subscript refers to the quadrilateral element with five nodes. Equation 17, partitioned to isolate the degrees of freedom of the fifth node can be written as

$$\begin{pmatrix} R_I \\ R_{II} \end{pmatrix} = \begin{pmatrix} k_{I,I} & k_{I,II} \\ k_{II,I} & k_{II,II} \end{pmatrix} \begin{pmatrix} r_I \\ r_{II} \end{pmatrix} \quad (18)$$

Equation 18 can be written as two separate equations

$$R_I = k_{I,I} r_I + k_{I,II} r_{II} \quad (19)$$

$$R_{II} = k_{II,I} r_I + k_{II,II} r_{II} \quad (20)$$

Since the fifth node doesn't actually exist in the original model, no external forces can be applied to this node. This condition gives

$$r_{II} = -k_{II,II}^{-1} k_{II,I} r_I \quad (21)$$

Substitution of equation 21 in equation 19 gives

$$R_I = (k_{I,I} - k_{I,II} k_{II,II}^{-1} k_{II,I}) r_I \quad (22)$$

From equation 22 the stiffness matrix of the original quadrilateral membrane element can be written as

$$\underline{k} = \underline{k}_{II} - \underline{k}_{I,II} \underline{k}_{II,II}^{-1} \underline{k}_{II,I} \quad (23)$$

The shear element is also composed of four triangular elements however, the stiffness matrices of the component triangles are determined by considering only the shear strain energy (equation 13).

$$k_{ij} = \frac{1}{2} |\underline{X}|^t \epsilon_{xy}^{(i)} G \epsilon_{xy}^{(j)} \quad (24)$$

MODAL STRAIN ENERGY PROGRAM

The program starts by reading in all the information it needs from a NASTRAN output file for a normal modes analysis. As the program is set up, it can handle 1,000 of any one type of element for a total of 6,000 elements. A total of 100 materials can be specified but only isotropic materials specified on MAT1 cards are currently accounted for. The model can have 100 properties for any one element type for a total of 600 property cards. These limits can easily be expanded by changing the dimensions of the arrays in the code. The CELAS elements (CELAS1 or CELAS2) must be grounded (fixed) at one end with the other end connected to the structure. There are two ways to do this, one is to leave the second grid point of the CELAS card and its component blank or the second way is to specify a second grid point and component and then fix the second grid point component with an SPC card.

The next step in determining the modal strain energies is to calculate the element stiffness matrices. Using the equations derived above and equations for the CELAS, CBAR, and CROD elements, the stiffness matrices are generated. After this the eigenvector for the current element is extracted from the eigenvector table for a given mode. Then the following equation is used to determine the element modal strain energies for the given mode

$$\text{Element Modal Strain Energy} = \frac{1}{2} \phi^r \underline{k} \phi^r \quad (25)$$

The equation is used for each element for every mode printed in the NASTRAN normal modes analysis.

After the element strain energies are calculated, they are printed in an easy to read format. The modal strain energy program prints out the following quantities for each mode: element ID number (EID), element type (CBAR, CELAS, CROD, CTRMEM, CSHEAR, or CQDMEM2), element strain energy (in consistent units), percent element strain energy of the entire structure, sum of the total element strain energy for each element type, and the total element strain energy for the entire structure. The program also prints one-half the generalized stiffness from the NASTRAN output file as a check. One-half the generalized stiffness should equal the total strain energy for the entire structure.

APPLICATIONS

Viscoelastic materials are seeing widespread use to suppress vibrations in all types of

structures. The ability of viscoelastic materials to passively damp vibrations in lightweight structures is well documented. Modal strain energies are useful in estimating the damping in this type of structure. The approach used to predict the modal damping (loss) factors for each mode of the structure is called the modal strain energy method. It states that the ratio of structural loss factor to viscoelastic material loss factor for a given mode of vibration can be estimated as the ratio of elastic strain energy in the viscoelastic to total elastic strain energy in the entire structure when it deforms into the particular undamped mode shape (ref. 3). Mathematically this can be stated as

$$\frac{\eta_s^{(r)}}{\eta_v} = \frac{V_v^{(r)}}{V_s^{(r)}} \quad (26)$$

where

$\eta_s^{(r)}$ = loss factor for the r 'th mode of the composite structure

η_v = material loss factor for the viscoelastic material

$V_v^{(r)}$ = elastic strain energy stored in the viscoelastic material when the structure deforms in its r 'th undamped mode shape

$V_s^{(r)}$ = elastic strain energy of the entire composite structure in the r 'th mode shape

Computing the undamped mode shapes of the composite structure with the viscoelastic material treated as if it were purely elastic with a real stiffness modulus, the right hand side of equation 26 is calculated as

$$\frac{V_v^{(r)}}{V_s^{(r)}} = \frac{\sum_{\theta=1}^n \phi_{\theta}^{r^t} k_{\theta} \phi_{\theta}^r}{\phi^{r^t} K \phi^r} \quad (27)$$

where

ϕ^r = r 'th mode shape vector

ϕ_{θ}^r = subvector formed by deleting from ϕ all entries not corresponding to motion of nodes of the θ 'th viscoelastic element

k_{θ} = element stiffness matrix of the θ 'th viscoelastic element

K = stiffness matrix of the entire composite structure

n = number of viscoelastic elements in the model

Combining equations 26 and 27 you get (ref. 4)

$$\eta_s^{(r)} = \frac{\sum_{\theta=1}^n \eta_{v_{\theta}} \phi_{\theta}^{r^t} k_{\theta} \phi_{\theta}^r}{\phi^{r^t} K \phi^r} \quad (28)$$

This equation states that if you create a NASTRAN model of the damped structure with all elements included except damper elements, and then run a normal modes analysis,

you have all the information needed to get the structural loss factor. After you make the NASTRAN run, you run the output through the element modal strain energy program which gives you the percentages of element strain energy to total strain energy for the entire structure. The percentages for the elements that actually possess viscoelastic damping are multiplied by that particular elements material loss factor. These quantities are then summed to give the loss factor for the entire structure.

EXAMPLE PROBLEMS

Three example problems were used to demonstrate the ability of the element strain energy program to accurately output the element modal strain energies. The first problem is a rectangular wing box and it is shown broken up into its numbered elements in figure 1. The rectangular wing box consists of quadrilateral membrane elements for the inboard top and bottom skins, triangular membrane elements for the outboard top and bottom skins, bar elements for the outboard posts, rod elements for all other posts, shear elements for all the ribs and spars, elastic elements provide the inboard top skin attachment points with the inboard bottom skin points rigidly fixed. The strain energy outputs for the second mode of this model are given in table I. This model contains all the element types the strain energy program is capable of handling. Comparing the total structural strain energy with the value for one-half the generalized stiffness shows that the two are in agreement.

The second example is known as the intermediate complexity wing and is just a simplified NASTRAN model of the load carrying portion of a wing. Shown in figure 2 broken into its component elements and their numbering scheme, is a depiction of the wing. The model consists of quadrilateral membrane elements for most of the top and bottom skins, two triangular membrane elements for the outboard corner elements on the top and bottom skins, rod elements for all the posts, and shear elements for all the ribs and spars. The inboard top and bottom skin points rigidly fixed. The strain energy program output for the first mode of the model is given in table II. As you can see, the program accurately produces zero strain energy in all the rod elements for the first bending mode of the wing. The difference in the values for the total structural strain energy and one-half the generalized stiffness can be attributed to the different formulations of the stiffness matrices of the plate membrane and shear elements.

The final example is a part of the Large Space Structures Technology Program at the Flight Dynamics Laboratory. A NASTRAN model of the twelve meter truss structure is shown in figure 3. The elements aren't numbered because of the large number of elements in the model. The model consists of bar elements for the horizontal and vertical elements and rod elements for all the diagonals. The diagonal members contain the viscoelastic dampers on the actual structure. The model is supported at the base with a series of elastic elements. The strain energy output for the second mode is given in table III. This is the first torsion mode of the truss, so most of the strain energy is in the diagonal members. This is verified by the modal strain energy program. The loss factor for the entire structure has been predicted and is awaiting test results for verification.

CONCLUDING REMARKS

A FORTRAN program that calculates element strain energies has been developed and

verified. This program gives COSMIC NASTRAN a capability that was only previously available for static analysis. Work is currently underway to develop DMAP instructions to calculate modal strain energies directly in NASTRAN. With the ever increasing trend toward lighter structures, damping materials will see increased use in all types of structures. A simple, accurate method, such as the modal strain energy method, to predict structural damping is essential.

REFERENCES

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2. The NASTRAN User's Manual, Jun. 1985.
3. Johnson, C.D., Kienholz, D.A., and Rogers, L.C., "Finite Element Prediction of Damping in Beams with Constrained Viscoelastic Layers," Shock and Vibration Bulletin, No. 50, Part 1, May 1981, pp. 71-82.
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TABLE I - RECTANGULAR WING BOX STRAIN ENERGIES

MODE 2

| Element ID | Element Type | Element Strain Energy | % Strain Energy |
|------------|--------------|-----------------------|-----------------|
| 50001 | CBAR | -0.4068E-06 | 0.0000 |
| 50002 | CBAR | 6.446 | 0.2810 |
| 50003 | CBAR | -0.6010E-06 | 0.0000 |
| 500 | CELAS | 0.2516 | 0.0110 |
| 502 | CELAS | 0.2749E-01 | 0.0012 |
| 503 | CELAS | 0.2810E-01 | 0.0012 |
| 504 | CELAS | 69.68 | 3.0378 |
| 505 | CELAS | 55.31 | 2.4114 |
| 506 | CELAS | 50.50 | 2.2015 |
| 507 | CELAS | 0.5442 | 0.0237 |
| 508 | CELAS | 0.9635E-01 | 0.0042 |
| 509 | CELAS | 0.1016 | 0.0044 |
| 50004 | CROD | 0.0000E+00 | 0.0000 |
| 50005 | CROD | 0.0000E+00 | 0.0000 |
| 50006 | CROD | 0.0000E+00 | 0.0000 |
| 50007 | CROD | 1.088 | 0.0475 |
| 50008 | CROD | 0.1927 | 0.0084 |
| 50009 | CROD | 0.2032 | 0.0089 |
| 10001 | CTRMEM | 117.4 | 5.1190 |
| 10002 | CTRMEM | 116.7 | 5.0897 |

| | | | |
|-------|--------|------------|--------|
| 10011 | CTRMEM | 137.4 | 5.9880 |
| 10012 | CTRMEM | 117.8 | 5.1372 |
| 20001 | CTRMEM | 115.1 | 5.0171 |
| 20002 | CTRMEM | 119.7 | 5.2185 |
| 20011 | CTRMEM | 132.1 | 5.7594 |
| 20012 | CTRMEM | 115.9 | 5.0520 |
| 10003 | CQDMEM | 218.9 | 9.5449 |
| 10004 | CQDMEM | 166.3 | 7.2488 |
| 20003 | CQDMEM | 200.6 | 8.7462 |
| 20004 | CQDMEM | 189.0 | 8.2413 |
| 30001 | CSHEAR | 29.60 | 1.2904 |
| 30002 | CSHEAR | 88.67 | 3.8657 |
| 30003 | CSHEAR | 34.38 | 1.4987 |
| 30004 | CSHEAR | 80.61 | 3.5140 |
| 30005 | CSHEAR | 72.90 | 3.1781 |
| 30006 | CSHEAR | 28.34 | 1.2355 |
| 40001 | CSHEAR | 12.29 | 0.5360 |
| 40002 | CSHEAR | 8.467 | 0.3691 |
| 40003 | CSHEAR | 0.6263E-02 | 0.0003 |
| 40004 | CSHEAR | 3.315 | 0.1445 |
| 40005 | CSHEAR | 3.752 | 0.1636 |
| 40006 | CSHEAR | 0.9805E-05 | 0.0000 |

STRAIN ENERGY IN CELAS ELEMENTS = 176.5

STRAIN ENERGY IN CBAR ELEMENTS = 6.446

STRAIN ENERGY IN CROD ELEMENTS = 1.484

STRAIN ENERGY IN CTRMEM ELEMENTS = 972.1

STRAIN ENERGY IN CQDMEM ELEMENTS = 774.9

STRAIN ENERGY IN CSHEAR ELEMENTS = 362.3

TOTAL STRAIN ENERGY = 2294.

GENERALIZED STIFFNESS/2 = 2294.

TABLE II - INTERMEDIATE COMPLEXITY WING STRAIN ENERGIES

MODE 1

| Element ID | Element Type | Element Strain Energy | % Strain Energy |
|------------|--------------|-----------------------|-----------------|
| 120 | CROD | 0.0000E+00 | 0.0000 |
| 121 | CROD | 0.0000E+00 | 0.0000 |
| 122 | CROD | 0.0000E+00 | 0.0000 |
| 123 | CROD | 0.0000E+00 | 0.0000 |
| 124 | CROD | 0.0000E+00 | 0.0000 |
| 125 | CROD | 0.0000E+00 | 0.0000 |
| 126 | CROD | 0.0000E+00 | 0.0000 |
| 127 | CROD | 0.0000E+00 | 0.0000 |
| 128 | CROD | 0.0000E+00 | 0.0000 |

| | | | |
|-----|--------|------------|--------|
| 129 | CROD | 0.0000E+00 | 0.0000 |
| 130 | CROD | 0.0000E+00 | 0.0000 |
| 131 | CROD | 0.0000E+00 | 0.0000 |
| 132 | CROD | 0.0000E+00 | 0.0000 |
| 133 | CROD | 0.0000E+00 | 0.0000 |
| 134 | CROD | 0.0000E+00 | 0.0000 |
| 135 | CROD | 0.0000E+00 | 0.0000 |
| 136 | CROD | 0.0000E+00 | 0.0000 |
| 137 | CROD | 0.0000E+00 | 0.0000 |
| 138 | CROD | 0.0000E+00 | 0.0000 |
| 139 | CROD | 0.0000E+00 | 0.0000 |
| 140 | CROD | 0.0000E+00 | 0.0000 |
| 141 | CROD | 0.0000E+00 | 0.0000 |
| 142 | CROD | 0.0000E+00 | 0.0000 |
| 143 | CROD | 0.0000E+00 | 0.0000 |
| 144 | CROD | 0.0000E+00 | 0.0000 |
| 145 | CROD | 0.0000E+00 | 0.0000 |
| 146 | CROD | 0.0000E+00 | 0.0000 |
| 147 | CROD | 0.0000E+00 | 0.0000 |
| 148 | CROD | 0.0000E+00 | 0.0000 |
| 149 | CROD | 0.0000E+00 | 0.0000 |
| 150 | CROD | 0.0000E+00 | 0.0000 |
| 151 | CROD | 0.0000E+00 | 0.0000 |
| 152 | CROD | 0.0000E+00 | 0.0000 |
| 153 | CROD | 0.0000E+00 | 0.0000 |
| 154 | CROD | 0.0000E+00 | 0.0000 |
| 155 | CROD | 0.0000E+00 | 0.0000 |
| 156 | CROD | 0.0000E+00 | 0.0000 |
| 157 | CROD | 0.0000E+00 | 0.0000 |
| 158 | CROD | 0.0000E+00 | 0.0000 |
| 1 | CTRMEM | 0.2817E-01 | 0.0046 |
| 2 | CTRMEM | 0.2817E-01 | 0.0046 |
| 3 | CQDMEM | 0.8132E-01 | 0.0131 |
| 4 | CQDMEM | 0.8132E-01 | 0.0131 |
| 5 | CQDMEM | 0.6809E-01 | 0.0110 |
| 6 | CQDMEM | 0.6809E-01 | 0.0110 |
| 7 | CQDMEM | 0.1561 | 0.0252 |
| 8 | CQDMEM | 0.1561 | 0.0252 |
| 9 | CQDMEM | 0.6461 | 0.1045 |
| 10 | CQDMEM | 0.6461 | 0.1045 |
| 11 | CQDMEM | 0.7805 | 0.1262 |
| 12 | CQDMEM | 0.7805 | 0.1262 |
| 13 | CQDMEM | 0.7629 | 0.1234 |
| 14 | CQDMEM | 0.7629 | 0.1234 |
| 15 | CQDMEM | 0.9026 | 0.1459 |
| 16 | CQDMEM | 0.9026 | 0.1459 |
| 17 | CQDMEM | 2.573 | 0.4161 |
| 18 | CQDMEM | 2.573 | 0.4161 |
| 19 | CQDMEM | 3.169 | 0.5124 |
| 20 | CQDMEM | 3.169 | 0.5124 |
| 21 | CQDMEM | 3.299 | 0.5335 |
| 22 | CQDMEM | 3.299 | 0.5335 |
| 23 | CQDMEM | 3.306 | 0.5345 |

| | | | |
|----|--------|------------|--------|
| 24 | CQDMEM | 3.306 | 0.5345 |
| 25 | CQDMEM | 5.762 | 0.9316 |
| 26 | CQDMEM | 5.762 | 0.9316 |
| 27 | CQDMEM | 7.520 | 1.2159 |
| 28 | CQDMEM | 7.520 | 1.2159 |
| 29 | CQDMEM | 7.967 | 1.2882 |
| 30 | CQDMEM | 7.967 | 1.2882 |
| 31 | CQDMEM | 7.312 | 1.1824 |
| 32 | CQDMEM | 7.312 | 1.1824 |
| 33 | CQDMEM | 9.504 | 1.5367 |
| 34 | CQDMEM | 9.504 | 1.5367 |
| 35 | CQDMEM | 12.89 | 2.0840 |
| 36 | CQDMEM | 12.89 | 2.0840 |
| 37 | CQDMEM | 14.20 | 2.2956 |
| 38 | CQDMEM | 14.20 | 2.2956 |
| 39 | CQDMEM | 12.59 | 2.0364 |
| 40 | CQDMEM | 12.59 | 2.0364 |
| 41 | CQDMEM | 12.67 | 2.0479 |
| 42 | CQDMEM | 12.67 | 2.0479 |
| 43 | CQDMEM | 17.60 | 2.8462 |
| 44 | CQDMEM | 17.60 | 2.8462 |
| 45 | CQDMEM | 21.24 | 3.4346 |
| 46 | CQDMEM | 21.24 | 3.4346 |
| 47 | CQDMEM | 19.38 | 3.1329 |
| 48 | CQDMEM | 19.38 | 3.1329 |
| 49 | CQDMEM | 13.58 | 2.1953 |
| 50 | CQDMEM | 13.58 | 2.1953 |
| 51 | CQDMEM | 17.25 | 2.7891 |
| 52 | CQDMEM | 17.25 | 2.7891 |
| 53 | CQDMEM | 20.66 | 3.3399 |
| 54 | CQDMEM | 20.66 | 3.3399 |
| 55 | CQDMEM | 16.06 | 2.5971 |
| 56 | CQDMEM | 16.06 | 2.5971 |
| 57 | CQDMEM | 6.645 | 1.0744 |
| 58 | CQDMEM | 6.645 | 1.0744 |
| 59 | CQDMEM | 17.18 | 2.7780 |
| 60 | CQDMEM | 17.18 | 2.7780 |
| 61 | CQDMEM | 21.31 | 3.4457 |
| 62 | CQDMEM | 21.31 | 3.4457 |
| 63 | CQDMEM | 23.16 | 3.7449 |
| 64 | CQDMEM | 23.16 | 3.7449 |
| 65 | CSHEAR | 0.4318E-01 | 0.0070 |
| 66 | CSHEAR | 0.5833E-04 | 0.0000 |
| 67 | CSHEAR | 0.4419E-01 | 0.0071 |
| 68 | CSHEAR | 0.6681E-03 | 0.0001 |
| 69 | CSHEAR | 0.1559E-02 | 0.0003 |
| 70 | CSHEAR | 0.3317E-01 | 0.0054 |
| 71 | CSHEAR | 0.3689E-02 | 0.0006 |
| 72 | CSHEAR | 0.1579E-01 | 0.0026 |
| 73 | CSHEAR | 0.3236E-02 | 0.0005 |
| 74 | CSHEAR | 0.6835E-02 | 0.0011 |
| 75 | CSHEAR | 0.3295E-02 | 0.0005 |
| 76 | CSHEAR | 0.7111E-02 | 0.0011 |

| | | | |
|-----|--------|------------|--------|
| 77 | CSHEAR | 0.3093E-02 | 0.0005 |
| 78 | CSHEAR | 0.4573E-02 | 0.0007 |
| 79 | CSHEAR | 0.2292E-02 | 0.0004 |
| 80 | CSHEAR | 0.4423E-02 | 0.0007 |
| 81 | CSHEAR | 0.3590E-02 | 0.0006 |
| 82 | CSHEAR | 0.4746E-02 | 0.0008 |
| 83 | CSHEAR | 0.1902E-02 | 0.0003 |
| 84 | CSHEAR | 0.4688E-02 | 0.0008 |
| 85 | CSHEAR | 0.7538E-02 | 0.0012 |
| 86 | CSHEAR | 0.8932E-02 | 0.0014 |
| 87 | CSHEAR | 0.3638E-02 | 0.0006 |
| 88 | CSHEAR | 0.9771E-02 | 0.0016 |
| 89 | CSHEAR | 0.9090E-01 | 0.0147 |
| 90 | CSHEAR | 0.6178E-01 | 0.0100 |
| 91 | CSHEAR | 0.4218E-01 | 0.0068 |
| 92 | CSHEAR | 0.6935E-01 | 0.0112 |
| 93 | CSHEAR | 0.5735 | 0.0927 |
| 94 | CSHEAR | 0.3672 | 0.0594 |
| 95 | CSHEAR | 0.1479 | 0.0239 |
| 96 | CSHEAR | 0.2381 | 0.0385 |
| 97 | CSHEAR | 0.2449 | 0.0396 |
| 98 | CSHEAR | 0.4526 | 0.0732 |
| 99 | CSHEAR | 0.5576 | 0.0902 |
| 100 | CSHEAR | 0.5448 | 0.0881 |
| 101 | CSHEAR | 0.3861 | 0.0624 |
| 102 | CSHEAR | 0.8999E-01 | 0.0146 |
| 103 | CSHEAR | 0.7155 | 0.1157 |
| 104 | CSHEAR | 0.1065 | 0.0172 |
| 105 | CSHEAR | 0.8149 | 0.1318 |
| 106 | CSHEAR | 1.047 | 0.1692 |
| 107 | CSHEAR | 1.125 | 0.1819 |
| 108 | CSHEAR | 1.076 | 0.1740 |
| 109 | CSHEAR | 1.026 | 0.1659 |
| 110 | CSHEAR | 0.9453 | 0.1528 |
| 111 | CSHEAR | 1.735 | 0.2805 |
| 112 | CSHEAR | 0.5387E-01 | 0.0087 |
| 113 | CSHEAR | 0.3877 | 0.0627 |
| 114 | CSHEAR | 0.6420 | 0.1038 |
| 115 | CSHEAR | 0.7546 | 0.1220 |
| 116 | CSHEAR | 0.7662 | 0.1239 |
| 117 | CSHEAR | 0.7926 | 0.1281 |
| 118 | CSHEAR | 0.5955 | 0.0963 |
| 119 | CSHEAR | 1.297 | 0.2097 |

STRAIN ENERGY IN CROD ELEMENTS = 0.0000E+00

STRAIN ENERGY IN CTRMEM ELEMENTS = 0.5633E-01

STRAIN ENERGY IN CQDMEM ELEMENTS = 600.4

STRAIN ENERGY IN CSHEAR ELEMENTS = 17.97

TOTAL STRAIN ENERGY = 618.5

GENERALIZED STIFFNESS/2 = 619.0

TABLE III - TWELVE METER TRUSS STRAIN ENERGIES

MODE 2

| Element ID | Element Type | Element Strain Energy | % Strain Energy |
|------------|--------------|--------------------------|-----------------|
| 101 | CBAR | 1.740 | 6.0085 |
| 102 | CBAR | 1.724 | 5.9537 |
| 103 | CBAR | 1.127 | 3.8917 |
| 104 | CBAR | 1.155 | 3.9862 |
| 106 | CBAR | 0.6792 | 2.3449 |
| 107 | CBAR | 0.6611 | 2.2827 |
| 108 | CBAR | 0.3441 | 1.1880 |
| 109 | CBAR | 0.3522 | 1.2160 |
| 111 | CBAR | 0.1476 | 0.5098 |
| 112 | CBAR | 0.1436 | 0.4956 |
| 113 | CBAR | 0.4434E-01 | 0.1531 |
| 114 | CBAR | 0.4531E-01 | 0.1564 |
| 116 | CBAR | 0.7187E-02 | 0.0248 |
| 117 | CBAR | 0.6948E-02 | 0.0240 |
| 118 | CBAR | 0.2278E-03 | 0.0008 |
| 119 | CBAR | 0.2049E-03 | 0.0007 |
| 120 | CBAR | 0.1767E-01 | 0.0610 |
| 121 | CBAR | 0.5047E-02 | 0.0174 |
| 122 | CBAR | 0.4326E-02 | 0.0149 |
| 123 | CBAR | 0.5652E-02 | 0.0195 |
| 125 | CBAR | 0.2255E-02 | 0.0078 |
| 126 | CBAR | 0.3096E-02 | 0.0107 |
| 127 | CBAR | 0.1182E-02 | 0.0041 |
| 128 | CBAR | 0.1833E-02 | 0.0063 |
| 130 | CBAR | 0.3633E-03 | 0.0013 |
| 131 | CBAR | 0.6576E-03 | 0.0023 |
| 132 | CBAR | 0.1058E-03 | 0.0004 |
| 133 | CBAR | 0.2597E-03 | 0.0009 |
| 135 | CBAR | 0.2277E-04 | 0.0001 |
| 136 | CBAR | 0.3004E-04 | 0.0001 |
| 137 | CBAR | 0.4012E-04 | 0.0001 |
| 138 | CBAR | 0.1046E-04 | 0.0000 |
| 139 | CBAR | 1.740 | 6.0085 |
| 140 | CBAR | 1.724 | 5.9537 |
| 141 | CBAR | 1.127 | 3.8917 |
| 142 | CBAR | 1.155 | 3.9862 |
| 144 | CBAR | 0.6792 | 2.3449 |
| 145 | CBAR | 0.6611 | 2.2827 |
| 146 | CBAR | 0.3441 | 1.1880 |
| 147 | CBAR | 0.3522 | 1.2160 |
| 149 | CBAR | 0.1476 | 0.5098 |
| 150 | CBAR | 0.1436 | 0.4956 |
| 151 | CBAR | 0.4434E-01 | 0.1531 |
| 152 | CBAR | 0.4531E-01 | 0.1564 |
| 154 | CBAR | 0.7187E-02 | 0.0248 |

| | | | |
|-----|------|------------|--------|
| 155 | CBAR | 0.6948E-02 | 0.0240 |
| 156 | CBAR | 0.2278E-03 | 0.0008 |
| 157 | CBAR | 0.2049E-03 | 0.0007 |
| 158 | CBAR | 0.1767E-01 | 0.0610 |
| 159 | CBAR | 0.5047E-02 | 0.0174 |
| 160 | CBAR | 0.4326E-02 | 0.0149 |
| 161 | CBAR | 0.5652E-02 | 0.0195 |
| 163 | CBAR | 0.2255E-02 | 0.0078 |
| 164 | CBAR | 0.3096E-02 | 0.0107 |
| 165 | CBAR | 0.1182E-02 | 0.0041 |
| 166 | CBAR | 0.1833E-02 | 0.0063 |
| 168 | CBAR | 0.3633E-03 | 0.0013 |
| 169 | CBAR | 0.6576E-03 | 0.0023 |
| 170 | CBAR | 0.1058E-03 | 0.0004 |
| 171 | CBAR | 0.2597E-03 | 0.0009 |
| 173 | CBAR | 0.2277E-04 | 0.0001 |
| 174 | CBAR | 0.3004E-04 | 0.0001 |
| 175 | CBAR | 0.4012E-04 | 0.0001 |
| 176 | CBAR | 0.1046E-04 | 0.0000 |
| 201 | CBAR | 0.1054E-03 | 0.0004 |
| 202 | CBAR | 0.9258E-03 | 0.0032 |
| 203 | CBAR | 0.6164E-03 | 0.0021 |
| 204 | CBAR | 0.6752E-03 | 0.0023 |
| 205 | CBAR | 0.5726E-03 | 0.0020 |
| 206 | CBAR | 0.5726E-03 | 0.0020 |
| 207 | CBAR | 0.5583E-03 | 0.0019 |
| 208 | CBAR | 0.5189E-03 | 0.0018 |
| 209 | CBAR | 0.4704E-03 | 0.0016 |
| 210 | CBAR | 0.3950E-03 | 0.0014 |
| 211 | CBAR | 0.3950E-03 | 0.0014 |
| 212 | CBAR | 0.3261E-03 | 0.0011 |
| 213 | CBAR | 0.2753E-03 | 0.0010 |
| 214 | CBAR | 0.2157E-03 | 0.0007 |
| 215 | CBAR | 0.1482E-03 | 0.0005 |
| 216 | CBAR | 0.1482E-03 | 0.0005 |
| 217 | CBAR | 0.8562E-04 | 0.0003 |
| 218 | CBAR | 0.4871E-04 | 0.0002 |
| 219 | CBAR | 0.1600E-04 | 0.0001 |
| 220 | CBAR | 0.8716E-05 | 0.0000 |
| 221 | CBAR | 0.1055E-03 | 0.0004 |
| 222 | CBAR | 0.9257E-03 | 0.0032 |
| 223 | CBAR | 0.6163E-03 | 0.0021 |
| 224 | CBAR | 0.6755E-03 | 0.0023 |
| 225 | CBAR | 0.5725E-03 | 0.0020 |
| 226 | CBAR | 0.5725E-03 | 0.0020 |
| 227 | CBAR | 0.5583E-03 | 0.0019 |
| 228 | CBAR | 0.5191E-03 | 0.0018 |
| 229 | CBAR | 0.4706E-03 | 0.0016 |
| 230 | CBAR | 0.3950E-03 | 0.0014 |
| 231 | CBAR | 0.3950E-03 | 0.0014 |
| 232 | CBAR | 0.3260E-03 | 0.0011 |
| 233 | CBAR | 0.2754E-03 | 0.0010 |
| 234 | CBAR | 0.2158E-03 | 0.0007 |

| | | | |
|-----|------|------------|--------|
| 235 | CBAR | 0.1482E-03 | 0.0005 |
| 236 | CBAR | 0.1482E-03 | 0.0005 |
| 237 | CBAR | 0.8558E-04 | 0.0003 |
| 238 | CBAR | 0.4871E-04 | 0.0002 |
| 239 | CBAR | 0.1596E-04 | 0.0001 |
| 240 | CBAR | 0.8653E-05 | 0.0000 |
| 241 | CBAR | 0.1054E-03 | 0.0004 |
| 242 | CBAR | 0.9258E-03 | 0.0032 |
| 243 | CBAR | 0.6164E-03 | 0.0021 |
| 244 | CBAR | 0.6752E-03 | 0.0023 |
| 245 | CBAR | 0.5726E-03 | 0.0020 |
| 246 | CBAR | 0.5726E-03 | 0.0020 |
| 247 | CBAR | 0.5583E-03 | 0.0019 |
| 248 | CBAR | 0.5189E-03 | 0.0018 |
| 249 | CBAR | 0.4704E-03 | 0.0016 |
| 250 | CBAR | 0.3950E-03 | 0.0014 |
| 251 | CBAR | 0.3950E-03 | 0.0014 |
| 252 | CBAR | 0.3261E-03 | 0.0011 |
| 253 | CBAR | 0.2753E-03 | 0.0010 |
| 254 | CBAR | 0.2157E-03 | 0.0007 |
| 255 | CBAR | 0.1482E-03 | 0.0005 |
| 256 | CBAR | 0.1482E-03 | 0.0005 |
| 257 | CBAR | 0.8562E-04 | 0.0003 |
| 258 | CBAR | 0.4871E-04 | 0.0002 |
| 259 | CBAR | 0.1600E-04 | 0.0001 |
| 260 | CBAR | 0.8716E-05 | 0.0000 |
| 261 | CBAR | 0.1055E-03 | 0.0004 |
| 262 | CBAR | 0.9257E-03 | 0.0032 |
| 263 | CBAR | 0.6163E-03 | 0.0021 |
| 264 | CBAR | 0.6755E-03 | 0.0023 |
| 265 | CBAR | 0.5725E-03 | 0.0020 |
| 266 | CBAR | 0.5725E-03 | 0.0020 |
| 267 | CBAR | 0.5583E-03 | 0.0019 |
| 268 | CBAR | 0.5191E-03 | 0.0018 |
| 269 | CBAR | 0.4706E-03 | 0.0016 |
| 270 | CBAR | 0.3950E-03 | 0.0014 |
| 271 | CBAR | 0.3950E-03 | 0.0014 |
| 272 | CBAR | 0.3260E-03 | 0.0011 |
| 273 | CBAR | 0.2754E-03 | 0.0010 |
| 274 | CBAR | 0.2158E-03 | 0.0007 |
| 275 | CBAR | 0.1482E-03 | 0.0005 |
| 276 | CBAR | 0.1482E-03 | 0.0005 |
| 277 | CBAR | 0.8558E-04 | 0.0003 |
| 278 | CBAR | 0.4871E-04 | 0.0002 |
| 279 | CBAR | 0.1596E-04 | 0.0001 |
| 280 | CBAR | 0.8652E-05 | 0.0000 |
| 301 | CBAR | 0.7572E-01 | 0.2614 |
| 302 | CBAR | 0.5778E-01 | 0.1995 |
| 303 | CBAR | 0.9422E-01 | 0.3253 |
| 304 | CBAR | 0.5765E-01 | 0.1990 |
| 305 | CBAR | 0.8604E-01 | 0.2971 |
| 306 | CBAR | 0.5167E-01 | 0.1784 |
| 307 | CBAR | 0.6932E-01 | 0.2393 |

| | | | |
|-----|------|------------|--------|
| 308 | CBAR | 0.4699E-01 | 0.1622 |
| 309 | CBAR | 0.5217E-01 | 0.1801 |
| 310 | CBAR | 0.3316E-01 | 0.1145 |
| 311 | CBAR | 0.3317E-01 | 0.1145 |
| 312 | CBAR | 0.2202E-01 | 0.0760 |
| 313 | CBAR | 0.1538E-01 | 0.0531 |
| 314 | CBAR | 0.7785E-02 | 0.0269 |
| 315 | CBAR | 0.3781E-02 | 0.0131 |
| 316 | CBAR | 0.8850E-03 | 0.0031 |
| 317 | CBAR | 0.7571E-01 | 0.2614 |
| 318 | CBAR | 0.5777E-01 | 0.1995 |
| 319 | CBAR | 0.9421E-01 | 0.3253 |
| 320 | CBAR | 0.5764E-01 | 0.1990 |
| 321 | CBAR | 0.8604E-01 | 0.2971 |
| 322 | CBAR | 0.5167E-01 | 0.1784 |
| 323 | CBAR | 0.6931E-01 | 0.2393 |
| 324 | CBAR | 0.4699E-01 | 0.1622 |
| 325 | CBAR | 0.5218E-01 | 0.1801 |
| 326 | CBAR | 0.3316E-01 | 0.1145 |
| 327 | CBAR | 0.3318E-01 | 0.1145 |
| 328 | CBAR | 0.2202E-01 | 0.0760 |
| 329 | CBAR | 0.1539E-01 | 0.0531 |
| 330 | CBAR | 0.7787E-02 | 0.0269 |
| 331 | CBAR | 0.3783E-02 | 0.0131 |
| 332 | CBAR | 0.8851E-03 | 0.0031 |
| 333 | CBAR | 0.7572E-01 | 0.2614 |
| 334 | CBAR | 0.5778E-01 | 0.1995 |
| 335 | CBAR | 0.9422E-01 | 0.3253 |
| 336 | CBAR | 0.5765E-01 | 0.1990 |
| 337 | CBAR | 0.8604E-01 | 0.2971 |
| 338 | CBAR | 0.5167E-01 | 0.1784 |
| 339 | CBAR | 0.6932E-01 | 0.2393 |
| 340 | CBAR | 0.4699E-01 | 0.1622 |
| 341 | CBAR | 0.5217E-01 | 0.1801 |
| 342 | CBAR | 0.3316E-01 | 0.1145 |
| 343 | CBAR | 0.3317E-01 | 0.1145 |
| 344 | CBAR | 0.2202E-01 | 0.0760 |
| 345 | CBAR | 0.1538E-01 | 0.0531 |
| 346 | CBAR | 0.7786E-02 | 0.0269 |
| 347 | CBAR | 0.3781E-02 | 0.0131 |
| 348 | CBAR | 0.8850E-03 | 0.0031 |
| 349 | CBAR | 0.7571E-01 | 0.2614 |
| 350 | CBAR | 0.5777E-01 | 0.1995 |
| 351 | CBAR | 0.9421E-01 | 0.3253 |
| 352 | CBAR | 0.5764E-01 | 0.1990 |
| 353 | CBAR | 0.8604E-01 | 0.2971 |
| 354 | CBAR | 0.5167E-01 | 0.1784 |
| 355 | CBAR | 0.6931E-01 | 0.2393 |
| 356 | CBAR | 0.4699E-01 | 0.1622 |
| 357 | CBAR | 0.5218E-01 | 0.1801 |
| 358 | CBAR | 0.3316E-01 | 0.1145 |
| 359 | CBAR | 0.3318E-01 | 0.1145 |
| 360 | CBAR | 0.2203E-01 | 0.0760 |

| | | | |
|-----|-------|------------|---------|
| 361 | CBAR | 0.1539E-01 | 0.0531 |
| 362 | CBAR | 0.7787E-02 | 0.0269 |
| 363 | CBAR | 0.3783E-02 | 0.0131 |
| 364 | CBAR | 0.8852E-03 | 0.0031 |
| 500 | CELAS | 0.3367E-03 | 0.0012 |
| 501 | CELAS | 0.2813E-05 | 0.0000 |
| 502 | CELAS | 0.3367E-03 | 0.0012 |
| 503 | CELAS | 0.2813E-05 | 0.0000 |
| 504 | CELAS | 0.3367E-03 | 0.0012 |
| 505 | CELAS | 0.2812E-05 | 0.0000 |
| 506 | CELAS | 0.3367E-03 | 0.0012 |
| 507 | CELAS | 0.2812E-05 | 0.0000 |
| 508 | CELAS | 0.1465E-16 | 0.0000 |
| 509 | CELAS | 0.4949E-10 | 0.0000 |
| 510 | CELAS | 0.1465E-16 | 0.0000 |
| 511 | CELAS | 0.4949E-10 | 0.0000 |
| 512 | CELAS | 4.794 | 16.5529 |
| 513 | CELAS | 0.7808E-10 | 0.0000 |
| 514 | CELAS | 4.794 | 16.5529 |
| 515 | CELAS | 0.7808E-10 | 0.0000 |
| 516 | CELAS | 0.9110E-02 | 0.0315 |
| 517 | CELAS | 0.8964E-02 | 0.0309 |
| 518 | CELAS | 0.9110E-02 | 0.0315 |
| 519 | CELAS | 0.8964E-02 | 0.0309 |
| 520 | CELAS | 0.9111E-02 | 0.0315 |
| 521 | CELAS | 0.8964E-02 | 0.0310 |
| 522 | CELAS | 0.9111E-02 | 0.0315 |
| 523 | CELAS | 0.8964E-02 | 0.0310 |

STRAIN ENERGY IN CELAS ELEMENTS = 9.662

STRAIN ENERGY IN CBAR ELEMENTS = 19.30

TOTAL STRAIN ENERGY = 28.96

GENERALIZED STIFFNESS/2 = 28.96

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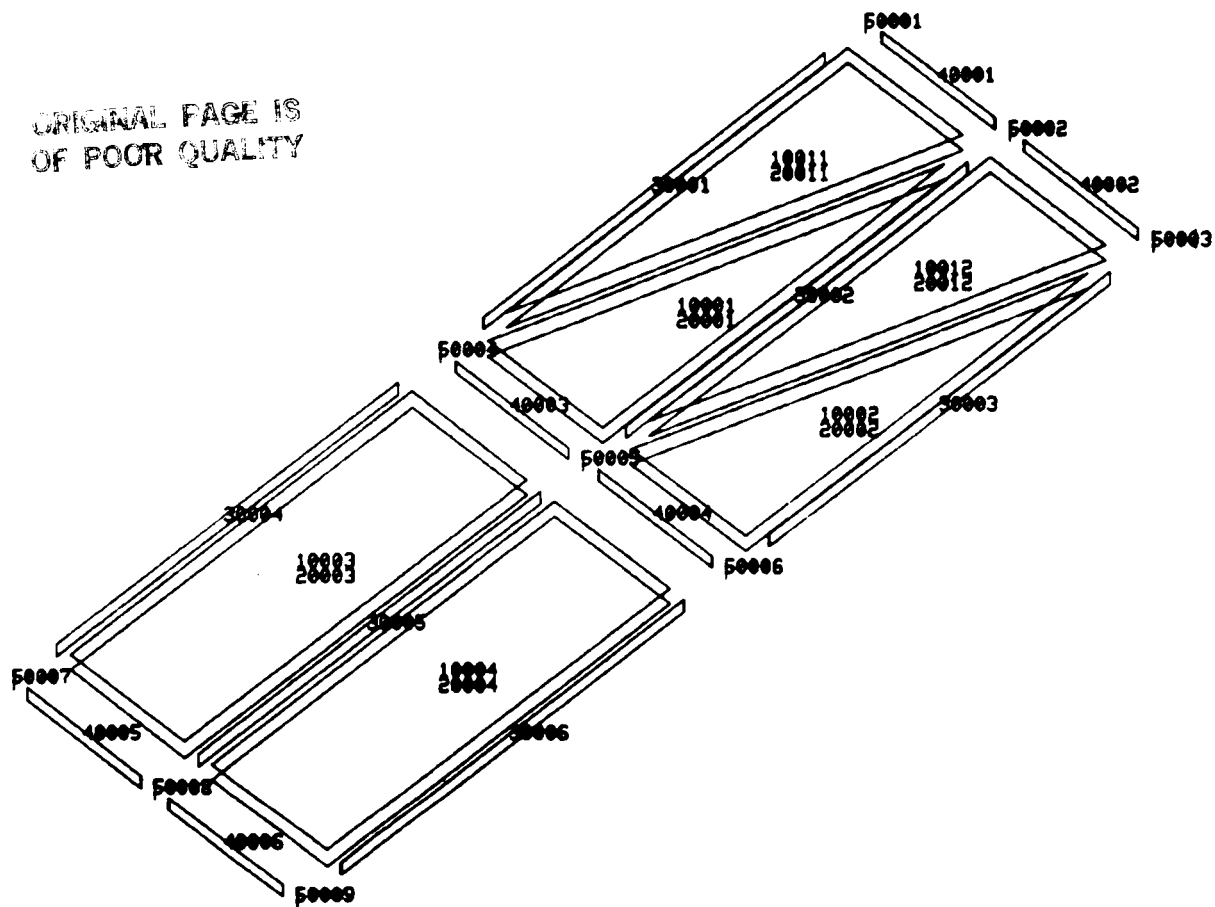


Figure 1 - Rectangular Wing Box Elements

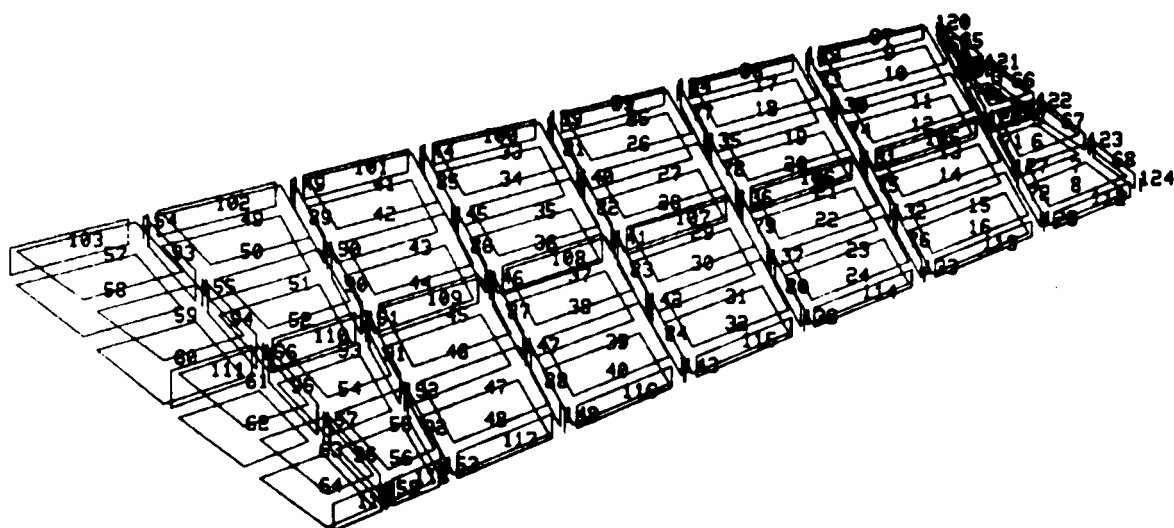


Figure 2 - Intermediate Complexity Wing Elements

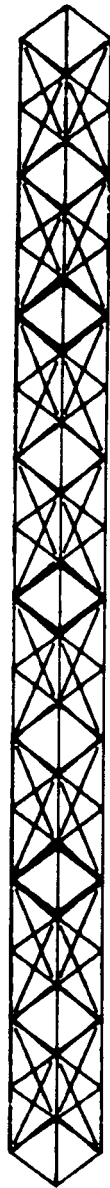


Figure 3 - Twelve Meter Truss Model